PH16212, Homework 1

Deadline: Sep. 25, 2019

1. For any four null vectors, p_i, p_j, p_k, p_l, we define the Parke-Taylor factor as

$$\mathrm{PT}(i,j,k,l) \equiv \frac{1}{\langle ij\rangle\langle jk\rangle\langle kl\rangle\langle li\rangle}\,. \tag{1}$$

Use Schouten identity to prove that

$$\mathrm{PT}(1,2,3,4)+\mathrm{PT}(1,3,2,4)+\mathrm{PT}(1,3,4,2)=0\,. \tag{2}$$

2. We have a gluon with the momentum $p^\mu=(1,0,0,1)$. $v^\mu=(1,1,0,0)$ is a "reference" vector. Use spinor helicity formalism, construct the $(+1)$-helicity polarization vector ϵ_+ and (-1)-helicity polarization vector ϵ_- such that

$$p\cdot\epsilon_\pm=0,\quad k\cdot\epsilon_\pm=0,\quad \epsilon_\pm^2=0,\quad \epsilon_+^\mu\epsilon_{-\mu}=-1\,. \tag{3}$$

Write down the Lorentz components of $\epsilon_\pm^\mu$.

3. We have a gluon with the momentum p_1^μ. Suppose that we used two different "reference" vectors, p_2 and p_3 ($p_2^2=0$, $p_3^2=0$), to construct the polarization vectors,

$$\epsilon_{1,+}^\mu=\sqrt{2}\frac{(\lambda_2\tilde\lambda_1)^\mu}{\langle 21\rangle},\quad \tilde\epsilon_{1,+}^\mu=\sqrt{2}\frac{(\lambda_3\tilde\lambda_1)^\mu}{\langle 31\rangle}, \tag{4}$$

Prove that

$$\epsilon_{1,+}^\mu-\tilde\epsilon_{1,+}^\mu=-\sqrt{2}\frac{\langle 23\rangle}{\langle 21\rangle\langle 31\rangle}\times p_1^\mu\,. \tag{5}$$

You need to use properties of Pauli matrices.

4. For four vector p_1,p_2,p_3,p_4, we define $\epsilon(1,2,3,4)\equiv\epsilon^{\mu_1\mu_2\mu_3\mu_4}p_{1\mu_1}p_{2\mu_2}p_{3\mu_3}p_{4\mu_4}$ where ϵ is the anti-symmetric tensor in 4D. We may further define that the Gram matrix as a 4×4 matrix $G(1,2,3,4)$,

$$G(1,2,3,4)_{ij}\equiv p_i\cdot p_j,\quad 1\le i,j\le 4\,. \tag{6}$$

Its determinant is $g(1,2,3,4)=\det G(1,2,3,4)$. Prove that

$$g(e_1,e_2,e_3,e_4)=-\epsilon(i,j,k,l)^2\,. \tag{7}$$

6. Consider the four-point massless kinematics, $p_1^2=0$, $p_2^2=0$, $p_3^2=0$, and $p_4^2=0$. Here $p_1+p_2+p_3+p_4=0$. Use momentum twistors to convert

$$\frac{[13][24]}{[14][23]} \tag{8}$$

to a function of s and t.